Chapter 15 - The Sun: A Nuclear Powerhouse

The energy radiated by the sun in one second is \(4 \times 10^{26}\) Joules. The rate energy is produced is called a Watt (Joules per second). How can the sun radiate energy at this rate and still be shining after \(4.6 \times 10^9\) years?

In the 19th century only two possible sources of energy were known: chemical and gravitational. Thermal energy is released in chemical reactions, such as oxidation, but the sun, even if it consisted completely of some flammable substance, would burn all of its mass in a few thousand years to release the observed amount of energy per second. Gravitational energy is a better source. As a sphere of gas contracts, gravitational potential energy can be converted into kinetic energy of the gas atoms. The increased speed of the atoms means an increase in temperature. A contracting
A ball of gas could remain hot enough to radiate the observed amount of energy persisting for around $1 \times 10^{9}$ years. This falls far short of the $4.5 \times 10^{9}$ year age of the solar system though.

In the 20th century, Einstein found that mass could theoretically be converted to energy. This happens in nuclear reactions. In fusion reactions, low mass nuclei can join to form heavier nuclei and release energy.

Let us examine a typical fusion reaction:

Symbolically:

$$^1H + ^1H \rightarrow ^2H + e^+ + \nu$$

What does this mean?

- $e^+$ positron from weak reaction converting proton to neutron
- $\nu$ electron neutrino created to conserve lepton number (no released)
- two protons collide form a deuterium nucleus (neutron + proton)
- releasing an anti-electron

Diagram:

1. Two protons collide
2. Form a deuterium nucleus (neutron + proton)
3. Releasing an anti-electron
Conservation Laws

Baryon number: proton + proton = 2
neutron + proton = 2
2 = 2.

Lepton number: 0 on left hand side
-1 for positron + 1 for neutron
= 0 on right hand side
0 = 0

Charge: proton + proton = +2 on left hand side
proton + neutron = +1
so positron + neutron = +1
so + 1 on right hand side
+2 = +2

The energy released can be computed from
\[ E = mc^2 \]
where \( c \) is the speed of light: \( c = 3 \times 10^8 \text{ m/s}, \ c^2 = 9 \times 10^{16} \text{ m}^2/\text{s}^2. \)

\[ \Delta E_{\text{bind}} = (9 \times 10^{-6}) \Delta m \text{ kg} \]
\[ \Delta m = \frac{1}{12} \text{ of } ^{12}\text{C mass} \]
\[ = 1.66054 \times 10^{-27} \text{ kg} \]
\[ = 1 \text{ atomic mass unit} \]

Example: Deuterium, \(^2\text{H}_2\), is composed of 1 proton
and 1 neutron.

In atomic mass units a proton masses 1,007385 u
and a neutron masses 1,008665 u

\[ \text{sum} \quad 2.016050 \]

A deuterium nucleus masses 2.01410 u

The difference in mass corresponds to an
energy difference. This is the Binding Energy

\[ \text{energy released} = \Delta E \approx (9 \times 10^{-6}) (2.01649 - 2.01410)(1.66054 \times 10^{-27}) \]
\[ = 3.57 \times 10^{-10} \text{ J} \]
or 2.23 MeV
compare to energy required to ionize a hydrogen atom, 13.6 eV.

\[
\frac{2.23 \text{ MeV}}{13.6 \text{ eV}} = \frac{2.23 \times 10^6 \text{ eV}}{13.6 \text{ eV}} = 164,150
\]

2.23 MeV is enough energy to ionize this many hydrogen atoms.

Another way to interpret this is the kinetic energy an electron has with 1 eV of energy: 1 eV corresponds with an electron temperature of ~11,000 K.

Nuclear reactions release a lot of energy!
Rule:
If the sum of the masses of the component nuclei is greater than the mass of the final nucleus, energy is released in the reaction: Mass decrease $\rightarrow$ energy.

Protons only combine if they get very close to one another. Since like charges repel, the kinetic energy (thus temperature!) must be high to overcome this repulsion. Nuclear reactions like this can only occur at high temperatures. (thermonuclear reactions)

The center of the Sun is ~15 million degrees Kelvin. Fusion occurs above ~7 million degrees, so in an inner region called the core of the Sun nuclear reactions generate energy from the release of binding energy when protons collide to form more complex nuclei. The core of the Sun contains roughly 40% of its mass. Surrounding the core, where temperatures are too low to allow nuclear reactions, there is an envelope of gas. Energy flows out of the core through the envelope until it reaches the surface of the photosphere.

[Diagram of Sun showing convective envelope in outer region and radiative envelope outside core]
The particular set of reactions that powers the Sun is called the proton-proton or p-p chain.

1. $^1_1H + ^1_1H \rightarrow ^2_1He + e^+ + \gamma$
   $e^+ + e^- \rightarrow 2\gamma$

2. $^2_1He + ^1_1H \rightarrow ^3_2He + \gamma$

3. $^3_2He + ^3_2He \rightarrow ^4_2He + 2(1\gamma)$
   Final result

Net reaction:

$^4_2He \rightarrow ^4_2He + 2e^+ + 2\gamma + 2\gamma$

Mases: 4.03660 4.00260

Four hydrogen nuclei combine to form one helium nucleus with a release of energy which comes out in the form of neutrinos and gamma rays.

As the gamma rays wend their way through the envelope they heat the gas and finally emerge from the photosphere as visible light.

$^4He$ is 0.7% lighter than $^4(1H)$ so...
the conversion of hydrogen to helium in the core releases energy enough to power the sun for $10^{10}$ years.

This solves the problem of where the Sun's energy comes from. The core will slowly change composition from 75% H and 25% He to a core of mostly He over billions of years.

Once most of the hydrogen is gone from the core the Sun will have evolved and nuclear burning will no longer occur in the core. The topic of stellar evolution and the late stages of a star's life will be treated later.

Aside: Rough lifetime of Sun —

\[
M_{\text{H, core}} = 1.989 \times 10^{30} \text{ kg} \\
40\% \text{ in core } \approx 7.96 \times 10^{29} \text{ kg} \\
\sim 75\% \text{ H}, \quad M_{\text{H}} = 5.97 \times 10^{30} \text{ kg of H} \\
0.71\% = 0.0071 \quad \text{converting all this H to He reduces the mass of the core by 0.71%} \\
\text{(change in mass of core)} = M_{\text{H}} \times 0.0071 = 4.24 \times 10^{27} \text{ kg} = \Delta M_{\text{H, core}} \\
E_{\text{release}} = (4 \times 10^{14}) \times (4.24 \times 10^{27} \text{ kg}) \\
\quad = 3.81 \times 10^{44} \text{ Joules over life of Sun} \\
\text{divide by } 4 \times 10^{26} \text{ J/s, luminosity of Sun} \\
\Rightarrow \text{Life of Sun } \sim 9.5 \times 10^{17} \text{ seconds } = 3 \times 10^{9} \text{ years}